Higgs bosons of a supersymmetric E_6 model at the Large Hadron Collider

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Abstract

It is found that CP symmetry may be explicitly broken in the Higgs sector of a supersymmetric E_6 model with two extra neutral gauge bosons at the one-loop level. The phenomenology of the model, the Higgs sector in particular, is studied for a reasonable parameter space of the model, in the presence of explicit CP violation at the one-loop level. At least one of the neutral Higgs bosons of the model might be produced via the WW fusion process at the Large Hadron Collider.

Keywords: Higgs Physics, Supersymmetric Standard Model

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I. Introduction

Anticipations among high energy physicists for the discovery of new physics at the Large Hadron Collider (LHC) are very high as it prepares to operate in full swing. There are a number of compelling rationales for anticipating new physics beyond the Standard Model (SM). One of them is the observed baryon asymmetry of the universe, which indicates the survival of more matter than antimater during the evolution of the universe. In the SM, the only source of CP violation is the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It has been noticed that the size of CP violation in the SM by means of the CKM matrix alone is too small to explain the observed value of the baryon-to-entropy ratio, $n_B/s \sim 8 \times 10^{-11}$ [1], if the universe had begun from a baryon-symmetric state. Thus, in order to explain the observed baryon asymmetry of the universe, other sources of adequate CP violation are required.

A number of alternative models beyond the SM have been investigated for the possibility of CP violations. Supersymmetry (SUSY) has been with us for several decades, which nowadays is regarded as the most certain candidate for new physics. In reality, the necessity of CP violation beyond the SM is not the only *raison d'etre* for the SUSY. There are multiples of arguments that support its existence in nature. Some supersymmetric models have also been studied in this context, as their sophisticated Higgs sectors may possess sources of CP violation [2]. For some phenomenologically realistic supersymmetric models extended from the SM, soft SUSY breaking terms are essential ingredients [3]. If these soft SUSY breaking terms contain complex phases, the phenomenological analyses of these supersymmetric standard models might not only be complicated but also involve CP violation.

The minimal supersymmetric standard model (MSSM) is the simplest version of supersymmetric extension of the SM. Its Higgs sector has two Higgs doublets in order to give masses to up-like quarks and down-like quarks separately. At the one-loop level, a complex phase in the soft SUSY breaking terms of the MSSM induces an explicit CP mixing between scalar and pseudoscalar Higgs bosons [4].

Non-minimal versions of supersymmetric extension of the SM have additional Higgs singlets and thus can dynamically solve the dimensional μ-parameter problem in the MSSM by means of the vacuum expectation value (VEV) of the Higgs singlet [5,6,7]. They have also been studied within the context of explicit CP violation in their Higgs sectors [8,9,10,11]. The next-to-minimal supersymmetric standard model (NMSSM) is a typical member of them. Unlike the MSSM, the Higgs potential of the NMSSM has one nontrivial CP phase after redefining the Higgs fields at the tree level [9]. At the one-loop level, it also develops CP violating phases. The effects of explicit CP violation at the one-loop level in the NMSSM on the masses of neutral and charged Higgs bosons are predicted in the literature [10].

The Higgs potentials of both the minimal non-minimal supersymmetric model and the $U(1)$-extended supersymmetric model may not have any CP phase at the tree level [11]. However, these models may also possess complex phases to induce explicit CP violation at the one-loop level, by taking the radiative corrections due to the quark and squark loops into account.

In this article, we would like to continue to study the possibility of CP violation in the Higgs sector of a supersymmetric E_6 model. This model has two $U(1)$ symmetries in addition to the SM gauge symmetry, thus with two additional neutral gauge bosons, and two Higgs singlets as well as two Higgs doublets [12,13]. The tree-level Higgs potential of this model may not have complex phase, because any complex phase can always be eliminated by rotating the relevant Higgs fields. At the one-loop level, it is shown that this model may allow CP violation in an explicit way due to radiative corrections. We study the Higgs phenomenology of this model by varying all the relevant parameters within reasonable ranges, to obtain the upper bound on the lightest neutral Higgs boson mass. We investigate prospects for discovering the neutral Higgs bosons of this model at the LHC, by calculating the minimum cross section for producing at least any one of the neutral Higgs bosons of this model via the WW fusion process at the LHC.

II. Higgs Sector

Let us describe the Higgs sector of our model. We assume that the electroweak gauge symmetry of our model is $SU(2) \times U(1) \times U_1(1) \times U_2(1)$, where the two extra $U(1)$ symmetries are decomposed from E_6. Thus, it is a kind of rank-6 supersymmetric model. We assume that in general $U_1(1)$ and $U_2(1)$ would mix with a certain mixing angle θ to become two linearly orthogonal combinations, $U(1)'$ and $U(1)''$. The Higgs sector of our model consists of two Higgs doublets, $H_1^T = (H_1^0, H_1^-)$ and $H_2^T = (H_2^+, H_2^0)$, and two neutral Higgs singlets, N_1 and N_2. The Yukawa interaction between Higgs superfields and quark superfields in the superpotential of our model may be expressed as [12,13]

$$\mathcal{W} \approx h_t Q^T \mathcal{H}_2 t_R^c - h_b Q^T \mathcal{H}_1 b_R^c + \lambda \mathcal{H}_1 \mathcal{H}_2 \mathcal{N}_1 \ , \tag{1}$$

where we take only the third generation into account and h_t and h_b are respectively the dimensionless Yukawa coupling coefficients of top and bottom quarks, λ is a dimensionless coefficient, $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Higgs doublet superfields, $\mathcal{N}$ is the Higgs singlet superfield, t_R^c and b_R^c are respectively the right-handed top and bottom quark superfields, and Q is the left-handed $SU(2)$ doublet quark superfield of the third generation. This superpotential has the same expression as discussed in Ref. [13] or Ref. [14], where relatively well-known rank-6 SUSY models are investigated.

Note that, shown as the last term in the superpotential, only N_1 participates in coupling to the Higgs doublets, because the underlying E_6 gauge symmetry does not allow the other Higgs singlet N_2 to do so [13]. Effectively, the coupling between N_1 and the Higgs doublets corresponds to the μ term in the MSSM where the μ-parameter is generated by the VEV of the N_1.

The Higgs potential of our model at the tree level is collected from D-terms, F-terms, and the soft terms in the superpotential. The most general form of the Higgs potential at the tree level is given as [13]

$$\begin{aligned} V_0 \ = \ & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 N_1^\dagger N_1 + m_4^2 N_2^\dagger N_2 - (\lambda A H_1 H_2 N_1 + \text{H.c.}) \\ & + |\lambda|^2 [H_1^\dagger H_1 H_2^\dagger H_2 + H_1^\dagger H_1 N_1^\dagger N_1 + H_2^\dagger H_2 N_1^\dagger N_1] \end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{g_2^2}{2}-|\lambda|^2\right)|H_1^\dagger H_2|^2+\frac{g_1^2+g_2^2}{8}(H_1^\dagger H_1-H_2^\dagger H_2)^2\\
&+\frac{g_1^{'2}}{72}[C_\theta(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1-5N_2^\dagger N_2)\\
&-\sqrt{15}S_\theta(H_1^\dagger H_1-N_1^\dagger N_1+N_2^\dagger N_2)]^2\\
&+\frac{g_1^{''2}}{72}[S_\theta(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1-5N_2^\dagger N_2)\\
&+\sqrt{15}C_\theta(H_1^\dagger H_1-N_1^\dagger N_1+N_2^\dagger N_2)]^2\ ,
\end{aligned}
\tag{2}
$$

where g_2, g_1, $g_1^{'}$, and $g_1^{''}$ are respectively the $SU(2)$, $U(1)$, $U(1)'$, and $U(1)''$ gauge coupling coefficients, A is a massive parameter, $C_\theta = \cos\theta$ and $S_\theta = \sin\theta$, and m_i $(i = 1, 2, 3, 4)$ are soft SUSY breaking masses. These four soft masses in the Higgs potential would eventually be eliminated by means of the minimum conditions for the Higgs potential with respect to four neutral Higgs fields.

The parameters of the Higgs potential are assumed to be generally complex. Thus, λ and A in the tree-level Higgs potential may be complex such that their complex phases may be factored out explicitly as $\lambda A e^{i\phi}$. We also assume that the VEVs, which four neutral components of the Higgs fields acquire after electroweak symmetry breaking, may in general be complex. However, by redefining the phases of H_1, H_2, and N_2, we may adjust the vacuum expectation values as $v_1 = \langle H_1^0\rangle$, $v_2 = \langle H_2^0\rangle$, $x_1 e^{i\phi_1} = \langle N_1\rangle$ and $x_2 = \langle N_2\rangle$, where v_1, v_2, x_1, and x_2 are real and the complex phase ϕ_1 is the overall phase in $\langle H_1 H_2 N_1\rangle$. Thus, looking at the Higgs potential at the tree level, one can easily notice that the only possible source of complex phases is $\lambda A H_1 H_2 N$. By further redefining the phase of the Higgs singlet N_1, it is always possible to make the two phases ϕ and ϕ_1 cancel each other so that the tree-level Higgs potential can be made completely real. Therefore, our model conserves the CP symmetry at the tree level.

After the electroweak symmetry breaking, the tree-level mass of top quark is given as $m_t^2 = (h_t v_2)^2$, and the tree-level masses of stop quarks are given by the on-shell Lagrangian as

$$
m^2_{\tilde{t}_1,\ \tilde{t}_2} = \frac{1}{2}(m_Q^2+m_T^2)+m_t^2+\frac{1}{4}m_Z^2\cos 2\beta+G_t^{'}\mp\sqrt{X_t}\ ,
\tag{3}
$$

where m_Q and m_T are the soft SUSY breaking masses for the stop quarks, $m_Z^2 = (g_1^2 + g_2^2)v^2/2$ with $v^2 = v_1^2 + v_2^2$ is the squared mass of the neutral weak gauge boson, $\tan\beta = v_2/v_1$, and

$$
\begin{aligned}
X_t &= \left(\frac{1}{2}(m_Q^2-m_T^2)+\left(\frac{2}{3}m_W^2-\frac{5}{12}m_Z^2\right)\cos 2\beta\right)^2\\
&\quad + m_t^2(A_t^2+\lambda^2 x_1^2\cot^2\beta-2\lambda A_t x_1\cot\beta\cos\phi_t)\ ,\\
G_t^{'} &= -\frac{g_1^{'2}}{4}\left(\frac{1}{3}\sqrt{\frac{5}{2}}S_\theta-\frac{1}{\sqrt{6}}C_\theta\right)\left[\left(\frac{\sqrt{10}}{3}S_\theta+\sqrt{\frac{2}{3}}C_\theta\right)v^2\cos^2\beta-\frac{2}{3}\sqrt{10}S_\theta x_1^2\right.\\
&\quad\left.+\left(\frac{\sqrt{10}}{3}S_\theta-\sqrt{\frac{2}{3}}C_\theta\right)v^2\sin^2\beta-\left(\frac{1}{3}\sqrt{\frac{5}{2}}S_\theta-\frac{5}{\sqrt{6}}C_\theta\right)x_2^2\right]\\
&\quad-\frac{g_1^{''2}}{4}\left(\frac{1}{3}\sqrt{\frac{5}{2}}C_\theta+\frac{1}{\sqrt{6}}S_\theta\right)\left[\left(\frac{\sqrt{10}}{3}C_\theta-\sqrt{\frac{2}{3}}S_\theta\right)v^2\cos^2\beta-\frac{2}{3}\sqrt{10}C_\theta x_1^2\right.
\end{aligned}
$$

$$+ \left(\frac{\sqrt{10}}{3} C_\theta + \sqrt{\frac{2}{3}} S_\theta \right) v^2 \sin^2 \beta - \left(\frac{1}{3} \sqrt{\frac{5}{2}} C_\theta + \frac{5}{\sqrt{6}} S_\theta \right) x_2^2 \Bigg] \ , \tag{4}$$

with $m_W^2 = g_2^2 v^2/2$ being the squared mass of the charged weak gauge boson, A_t being the trilinear soft SUSY breaking parameter of the stop quarks with mass dimension, and ϕ_t being a complex phase determined by ϕ_1 and the complex phase of A_t. Note that G_t' is the effect of the two extra $U(1)$ symmetries, but it does not contribute the mass splitting between the two stop quark masses. The mixing, and hence the mass splitting, between the stop quark masses is triggered by X_t.

Now let us consider the one-loop radiative corrections to the tree-level Higgs potential. In supersymmetric models, the incomplete cancellation between ordinary particles and their superpartners yield the one-loop corrections to the tree-level Higgs boson masses. In SUSY models, the most dominant part of the one-loop corrections to the tree-level Higgs potential come primarily from the top and stop quark loops. For large $\tan\beta$ as large as 50, the contribution of the bottom and sbottom quark loops can also be large. In this paper, we consider the contributions from the top and stop quark loops at the one-loop level. The full Higgs potential at the one-loop level may be written as $V = V_0 + V_1$, where V_1 is contribution from the radiative corrections due to the top and stop quark loops. The effective potential method provides us [15]

$$V_1 = \sum_l \frac{n_l \mathcal{M}_l^4}{64\pi^2} \left[\log \frac{\mathcal{M}_l^2}{\Lambda^2} - \frac{3}{2} \right] \ , \tag{5}$$

where Λ is the renormalization scale in the modified minimal subtraction scheme, the subscript l stands for the top and stop quarks: t, $\tilde{t}_1$, $\tilde{t}_2$, $\mathcal{M}_i$ are the top and stop masses at the tree level given as functions of Higgs fields, and n_i are the degrees of freedom for these particles. Including the sign convention, they are given as $n_t = -12$ and $n_{\tilde{t}_i} = 6$ $(i = 1, 2)$, as in the above formula enter the stop quarks with a negative sign while the top quark with a positive sign.

Since the parameters of the Higgs potential are assumed to be generally complex, we may have ϕ, which is the phase of λA. Unlike the tree-level case, we cannot redefine the phase of N_1 at the one-loop level to cancel it. Thus, ϕ may persist at the one-loop level. This can be most clearly be seen in the non-trivial tadpole minimum condition with respect to the pseudoscalar component of the Higgs field:

$$0 = A \sin\phi - \frac{3 m_t^2 A_t \sin\phi_t}{16\pi^2 v^2 \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \ , \tag{6}$$

where the first term comes from the tree-level Higgs potential and the second term comes from the radiative corrections, and the dimensionless function f arising from radiative corrections is defined as

$$f(m_x^2,\ m_y^2) = \frac{1}{(m_y^2 - m_x^2)} \left[m_x^2 \log \frac{m_x^2}{\Lambda^2} - m_y^2 \log \frac{m_y^2}{\Lambda^2} \right] + 1 \ . \tag{7}$$

But for the radiative corrections, the above tadpole minimum condition at the tree level would be satisfied when $\phi = 0$. Due to the presence of the one-loop corrections, $\phi = 0$ is no longer in general the solution to the above tadpole minimum condition.

Our model has twelve real degrees of freedom in the Higgs sector. They may be classified as three neutral Goldstone bosons, a pair of charged Goldstone bosons, five neutral Higgs bosons and a pair of charged Higgs bosons. After the electroweak symmetry breaking, the three neutral Goldstone bosons and a pair of charged Goldstone bosons will be absorbed into the longitudinal component of Z, Z', Z'' and W gauge bosons, where Z' and Z'' are the extra neutral gauge bosons.

The squared mass matrix M of the five neutral Higgs bosons is given as a symmetric 5×5 matrix, obtained by the second derivatives of the Higgs potential with respect to the five neutral Higgs fields. At the tree level, the five neutral Higgs bosons may have definite CP parity, since the CP symmetry is conserved in the Higgs sector. Thus, we may denote them as S_i $(i = 1, 2, 3, 4)$ for neutral scalar Higgs bosons and P for neutral pseudoscalar Higgs boson. In the (S_1, S_2, S_3, S_4, P) basis, the 5×5 matrix M at the tree level may be expressed as

$$M = M^0 + M^{0'} \ , \tag{8}$$

where $M^{0'}$ comes from the D-terms due to two extra $U(1)$ symmetries of V^0, and M^0 comes from the remaining terms in V_0, namely, D-terms due to the SM gauge symmetry, the F-terms, and the soft terms of the tree-level Higgs potential. They may be expressed as

$$M^0 = \begin{pmatrix} M^0_{11} & M^0_{12} & M^0_{13} & 0 & 0 \\ M^0_{12} & M^0_{22} & M^0_{23} & 0 & 0 \\ M^0_{13} & M^0_{23} & M^0_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M^0_{55} \end{pmatrix} \ , \tag{9}$$

$$M^{0'} = \begin{pmatrix} M^{0'}_{11} & M^{0'}_{12} & M^{0'}_{13} & M^{0'}_{14} & 0 \\ M^{0'}_{12} & M^{0'}_{22} & M^{0'}_{23} & M^{0'}_{24} & 0 \\ M^{0'}_{13} & M^{0'}_{23} & M^{0'}_{33} & M^{0'}_{34} & 0 \\ M^{0'}_{14} & M^{0'}_{24} & M^{0'}_{34} & M^{0'}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \ , \tag{10}$$

Explicitly, the matrix elements of M^0 and $M^{0'}$ are respectively given as follows:

$$\begin{aligned} M^0_{11} &= m_Z^2 \cos^2\beta + M^0_{55} \sin^2\beta \cos^2\alpha \ , \\ M^0_{22} &= m_Z^2 \sin^2\beta + M^0_{55} \cos^2\beta \cos^2\alpha \ , \\ M^0_{33} &= M^0_{55} \sin^2\alpha \ , \\ M^0_{12} &= (\lambda^2 v^2 - m_Z^2/2) \sin 2\beta - M^0_{55} \cos\beta \sin\beta \cos^2\alpha \ , \\ M^0_{13} &= 2\lambda^2 v x_1 \cos\beta - M^0_{55} \sin\beta \cos\alpha \sin\alpha \ , \\ M^0_{23} &= 2\lambda^2 v x_1 \sin\beta - M^0_{55} \cos\beta \cos\alpha \sin\alpha \ , \\ M^0_{55} &= 2\lambda A v \frac{\cos\phi}{\sin 2\alpha} \ , \end{aligned} \tag{11}$$

and

$$M^{0'}_{11} = \frac{1}{18}(g_1'^2 C_\theta^2 + g_1''^2 S_\theta^2) v^2 \cos^2\beta + \frac{5}{6}(g_1'^2 S_\theta^2 + g_1''^2 C_\theta^2) v^2 \cos^2\beta$$

$$
\begin{aligned}
& \quad - \frac{\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v^2 \cos^2\beta \ , \\
M_{22}^{0'} &= \frac{8}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v^2 \sin^2\beta \ , \\
M_{33}^{0'} &= \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_1^2 + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_1^2 - \frac{5\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta x_1^2 \ , \\
M_{44}^{0'} &= \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_2^2 + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_2^2 + \frac{5\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta x_2^2 \ , \\
M_{12}^{0'} &= \frac{1}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v^2 \sin 2\beta - \frac{\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v^2 \sin 2\beta \ , \\
M_{13}^{0'} &= - \frac{5}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_1 \cos\beta - \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) v x_1 \cos\beta \\
& \quad + \frac{\sqrt{15}}{3}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_1 \cos\beta \ , \\
M_{14}^{0'} &= - \frac{5}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_2 \cos\beta + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) v x_2 \cos\beta \\
& \quad + \frac{2\sqrt{15}}{3}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_2 \cos\beta \ , \\
M_{23}^{0'} &= - \frac{10}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_1 \sin\beta + \frac{2\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_1 \sin\beta \ , \\
M_{24}^{0'} &= - \frac{10}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_2 \sin\beta - \frac{2\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_2 \sin\beta \ , \\
M_{34}^{0'} &= \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_1 x_2 - \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_1 x_2 \ , \qquad (12)
\end{aligned}
$$

where $\tan\alpha = (v/2x_1)\sin 2\beta$ stands for the splitting between an extra $U(1)$ symmetry breaking scale and the electroweak scale.

Note that both M^0 and $M^{0'}$ do not mix S_i with P. In other words, there is no scalar-psuedoscalar mixing at the tree-level, hence the CP symmetry. It is straightforward to recognize that the single element M_{55}^0 is the squared mass at the tree level of the neutral pseudoscalar Higgs boson. Note also that if the two extra $U(1)$ symmetries are absent, we would have $M^{0'} = 0$. In this case, one of the neutral scalar Higgs bosons would be massless at the tree level, since M^0 may be decomposed into a block diagram consisting of three blocks, namely, 3×3 submatrix, $M_{44}^0 = 0$ and M_{55}^0.

Now, at the one-loop level, the squared mass matrix M of the five neutral Higgs bosons is corrected as

$$
M = M^0 + M^{0'} + M^1 \ , \qquad (13)
$$

where M^1 is the radiative corrections obtained from V^1 as

$$
M^1 = \left(\begin{array}{ccccc}
M_{11}^1 & M_{12}^1 & M_{13}^1 & M_{14}^1 & M_{15}^1 \\
M_{12}^1 & M_{22}^1 & M_{23}^1 & M_{24}^1 & M_{25}^1 \\
M_{13}^1 & M_{23}^1 & M_{33}^1 & M_{34}^1 & M_{35}^1 \\
M_{14}^1 & M_{24}^1 & M_{34}^1 & M_{44}^1 & M_{45}^1 \\
M_{15}^1 & M_{25}^1 & M_{35}^1 & M_{45}^1 & M_{55}^1
\end{array} \right) \ . \qquad (14)
$$

Explicitly, the matrix elements of M^1 are given as follows, after imposing tadpole minimum conditions:

$$
\begin{aligned}
M^1_{11} &= m_A^2 \sin^2\beta\cos^2\alpha - \frac{3\cos^2\beta}{16\pi^2 v^2}\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2 f(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2) \\
&\quad + \frac{3}{8\pi^2 v^2}\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta}+\frac{\cos\beta\Delta_{\tilde t}}{2}\right)^2 \frac{g(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2} \\
&\quad + \frac{3\cos^2\beta}{128\pi^2 v^2}(4G_a v^2+m_Z^2)^2\log\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{\Lambda^4}\right) \\
&\quad + \frac{3\cos\beta}{16\pi^2 v^2}(4G_a v^2+m_Z^2)\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta}+\frac{\cos\beta\Delta_{\tilde t}}{2}\right)\frac{\log(m_{\tilde t_2}^2/m_{\tilde t_1}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)}\ , \\
M^1_{22} &= m_A^2\cos^2\beta\cos^2\alpha - \frac{3\sin^2\beta}{16\pi^2 v^2}\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2 f(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2) \\
&\quad + \frac{3\sin^2\beta}{8\pi^2 v^2}\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin^2\beta}+\frac{\Delta_{\tilde t}}{2}\right)^2\frac{g(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2} - \frac{3m_t^4}{4\pi^2 v^2\sin^2\beta}\log\left(\frac{m_t^2}{\Lambda^2}\right) \\
&\quad - \frac{3\sin^2\beta}{16\pi^2 v^2}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2+4G_b v^2\right)\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin^2\beta}+\frac{\Delta_{\tilde t}}{2}\right)\frac{\log(m_{\tilde t_2}^2/m_{\tilde t_1}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)} \\
&\quad + \frac{3\sin^2\beta}{32\pi^2 v^2}\left(\frac{2m_t^2}{\sin^2\beta}-\frac{m_Z^2}{2}+2G_b v^2\right)^2\log\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{\Lambda^4}\right)\ , \\
M^1_{33} &= m_A^2\sin^2\alpha + \frac{3m_t^4\lambda^2\Delta_{\tilde t_1}^2}{8\pi^2\tan^2\beta}\frac{g(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2} - \frac{3G_c m_t^2 x_1\lambda\Delta_{\tilde t_1}}{4\pi^2\tan\beta}\frac{\log(m_{\tilde t_2}^2/m_{\tilde t_1}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)} \\
&\quad + \frac{3G_c^2 x_1^2}{8\pi^2}\log\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{\Lambda^4}\right)\ , \\
M^1_{44} &= \frac{3G_d^2 x_2^2}{8\pi^2}\log\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{\Lambda^4}\right)\ , \\
M^1_{55} &= m_A^2 + \frac{3m_t^4\lambda^2 A_t^2 x_1^2\sin^2\phi_t}{8\pi^2 v^2\sin^4\beta\cos^2\alpha}\frac{g(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2}\ , \\
M^1_{12} &= -m_A^2\cos\beta\sin\beta\cos^2\alpha + \frac{3\sin 2\beta}{32\pi^2 v^2}\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2 f(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2) \\
&\quad - \frac{3\sin\beta}{8\pi^2 v^2}\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta}+\frac{\cos\beta\Delta_{\tilde t}}{2}\right)\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin^2\beta}+\frac{\Delta_{\tilde t}}{2}\right)\frac{g(m_{\tilde t_1}^2,\ m_{\tilde t_2}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2} \\
&\quad + \frac{3\sin 2\beta}{32\pi^2 v^2}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2+4G_b v^2\right)\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin 2\beta}+\frac{\Delta_{\tilde t}}{4}\right)\frac{\log(m_{\tilde t_2}^2/m_{\tilde t_1}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)} \\
&\quad - \frac{3\sin 2\beta}{64\pi^2 v^2}(4G_a v^2+m_Z^2)\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin^2\beta}+\frac{\Delta_{\tilde t}}{2}\right)\frac{\log(m_{\tilde t_2}^2/m_{\tilde t_1}^2)}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)} \\
&\quad + \frac{3\sin 2\beta}{256\pi^2 v^2}(4G_a v^2+m_Z^2)\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2+4G_b v^2\right)\log\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{\Lambda^4}\right)\ ,
\end{aligned}
$$

$$
\begin{aligned}
M^1_{13} &= -m_A^2 \sin\beta\cos\alpha\sin\alpha - \frac{3m_t^2\lambda^2 x_1\cos\beta}{8\pi^2 v\sin^2\beta} f(m^2_{\tilde t_1},\ m^2_{\tilde t_2}) \\
&+ \frac{3m_t^2\lambda\Delta_{\tilde t_1}}{8\pi^2 v\tan\beta}\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta} + \frac{\cos\beta\Delta_{\tilde t}}{2}\right)\frac{g(m^2_{\tilde t_1},\ m^2_{\tilde t_2})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2} \\
&+ \frac{3m_t^2\lambda\cos\beta\Delta_{\tilde t_1}}{32\pi^2 v\tan\beta}(4G_a v^2 + m_Z^2)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&+ \frac{3G_c x_1}{8\pi^2 v}\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta} + \frac{\cos\beta\Delta_{\tilde t}}{2}\right)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&+ \frac{3G_c x_1\cos\beta}{32\pi^2 v}(4G_a v^2 + m_Z^2)\log\left(\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{\Lambda^4}\right)\ , \\
M^1_{14} &= \frac{3G_d x_2}{8\pi^2 v}\left(\frac{m_t^2\lambda x_1\Delta_{\tilde t_1}}{\sin\beta} + \frac{\cos\beta\Delta_{\tilde t}}{2}\right)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&+ \frac{3G_d x_2\cos\beta}{32\pi^2 v}(4G_a v^2 + m_Z^2)\log\left(\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{\Lambda^4}\right)\ , \\
M^1_{15} &= \frac{3m_t^4\lambda^2 A_t x_1^2\Delta_{\tilde t_1}\sin\phi_t}{8\pi^2 v^2\sin^3\beta\cos\alpha}\frac{g(m^2_{\tilde t_1},\ m^2_{\tilde t_2})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2} \\
&+ \frac{3m_t^2\lambda A_t\cos\beta\Delta_{\tilde t}\sin\phi_t}{16\pi^2 v\tan\beta\sin\alpha}\frac{g(m^2_{\tilde t_1},\ m^2_{\tilde t_2})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2} \\
&- \frac{3m_t^2\lambda A_t\cos\beta\sin\phi_t}{32\pi^2 v\tan\beta\sin\alpha}(4G_a v^2 + m_Z^2)\ \frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})}\ , \\
M^1_{23} &= -m_A^2\cos\beta\cos\alpha\sin\alpha \\
&- \frac{3m_t^2\lambda\Delta_{\tilde t_1}}{8\pi^2 v\tan\beta}\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin\beta} + \frac{\sin\beta\Delta_{\tilde t}}{2}\right)\frac{g(m^2_{\tilde t_1},\ m^2_{\tilde t_2})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2} \\
&+ \frac{3m_t^2\lambda\cos\beta\Delta_{\tilde t_1}}{16\pi^2 v}\left(\frac{2m_t^2}{\sin^2\beta} - \frac{m_Z^2}{2} + 2G_b v^2\right)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&- \frac{3G_c x_1}{8\pi^2 v}\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin\beta} + \frac{\sin\beta\Delta_{\tilde t}}{2}\right)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&+ \frac{3G_c x_1\sin\beta}{32\pi^2 v}\left(\frac{4m_t^2}{\sin^2\beta} + 4G_b v^2 - m_Z^2\right)\log\left(\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{\Lambda^4}\right)\ , \\
M^1_{24} &= -\frac{3G_d x_2}{8\pi^2 v}\left(\frac{m_t^2 A_t\Delta_{\tilde t_2}}{\sin\beta} + \frac{\sin\beta\Delta_{\tilde t}}{2}\right)\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\
&+ \frac{3G_d x_2\sin\beta}{32\pi^2 v}\left(\frac{4m_t^2}{\sin^2\beta} + 4G_b v^2 - m_Z^2\right)\log\left(\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{\Lambda^4}\right)\ , \\
M^1_{25} &= -\frac{3m_t^4\lambda A_t^2 x_1\Delta_{\tilde t_2}\sin\phi_t}{8\pi^2 v^2\sin^3\beta\cos\alpha}\frac{g(m^2_{\tilde t_1},\ m^2_{\tilde t_2})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2}
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{3m_t^2 \lambda A_t \cos\beta \Delta_{\tilde{t}} \sin\phi_t}{16\pi^2 v \sin\alpha} \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} \\
& + \frac{3m_t^2 \lambda A_t \cos\beta \sin\phi_t}{32\pi^2 v \sin\alpha} \left(\frac{4m_t^2}{\sin^2\beta} + 4G_b v^2 - m_Z^2 \right) \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \ , \\
M_{34}^1 \ &= \ \frac{3m_t^2 G_d x_2 \lambda \Delta_{\tilde{t}_1}}{8\pi^2 \tan\beta} \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} + \frac{3G_c G_d x_1 x_2}{8\pi^2} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{\Lambda^4} \right) \ , \\
M_{35}^1 \ &= \ \frac{3m_t^4 \lambda^2 A_t x_1 \Delta_{\tilde{t}_1} \sin\phi_t}{8\pi^2 v \sin^2\beta \tan\beta \cos\alpha} \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} \\
& + \frac{3m_t^2 G_c A_t \lambda v \cos^2\beta \sin\phi_t}{8\pi^2 \tan\alpha \sin\alpha} \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \ , \\
M_{45}^1 \ &= \ \frac{3m_t^2 G_d A_t x_2 \lambda \sin\phi_t}{8\pi^2 \tan\beta \sin\alpha} \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \ ,
\end{aligned} \tag{15}
$$

where

$$
m_A^2 = - \frac{3\lambda m_t^2 A_t \cos\phi_t}{8\pi^2 v \sin 2\alpha \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \ , \tag{16}
$$

$$
\begin{aligned}
\Delta_{\tilde{t}_1} \ &= \ \lambda x \cot\beta - A_t \cos\phi_t \ , \\
\Delta_{\tilde{t}_2} \ &= \ \lambda x \cot\beta \cos\phi_t - A_t \ , \\
\Delta_{\tilde{t}} \ &= \ \left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right) \left\{ (m_Q^2 - m_T^2) + \left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right) \cos 2\beta \right\} \ ,
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
G_a \ &= \ \frac{g_1^{'2}}{36}(4C_{2\theta} - 1) - \frac{g_1^{''2}}{36}(4C_{2\theta} + 1) \ , \\
G_b \ &= \ \frac{g_1^{'2}}{36}(\sqrt{15}S_{2\theta} + C_{2\theta} - 4) - \frac{g_1^{''2}}{36}(\sqrt{15}S_{2\theta} + C_{2\theta} + 4) \ , \\
G_c \ &= \ - \frac{g_1^{'2}}{18}(\sqrt{15}C_\theta - 5S_\theta)S_\theta + \frac{g_1^{''2}}{18}(\sqrt{15}S_\theta + 5C_\theta)C_\theta \ , \\
G_d \ &= \ \frac{g_1^{'2}}{72}(10 - 3\sqrt{15}S_{2\theta} + 5C_{2\theta}) + \frac{g_1^{''2}}{72}(10 + 3\sqrt{15}S_{2\theta} - 5C_{2\theta}) \ ,
\end{aligned} \tag{18}
$$

and the dimensionless function g is defined as

$$
g(m_x^2, m_y^2) = \frac{m_y^2 + m_x^2}{m_x^2 - m_y^2} \log \frac{m_y^2}{m_x^2} + 2 \ . \tag{19}
$$

Note first that the matrix elements M_{i5}^1 $(i = 1, 2, 3, 4)$ are proportional to $\sin\phi_t$. If $\phi_t = 0$, these elements would be zero, and the scalar-pseudoscalar mixing at the one-loop level would not occur in the Higgs sector. Therefore, there would be no CP violation in

the Higgs sector at the one-loop level. The squared mass of the pseudoscalar Higgs boson at the one-loop level, m_P^2, would be given simply by the $(5,5)$-th element of the M, taking $\phi_t = 0$. It is given by adding the radiative corrections as

$$m_P^2 = 2\lambda A v \frac{\cos\phi}{\sin 2\alpha} - \frac{3\lambda m_t^2 A_t}{8\pi^2 v \sin 2\alpha \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ . \tag{20}$$

In this case, the D-terms of extra $U(1)$ symmetries would not contribute to the mass of the pseudoscalar Higgs boson either at the tree level or at the one-loop level.

If, on the other hand, $\phi_t \neq 0$, there would be CP violation at the one-loop level, The CP phase in the radiative corrections generates the scalar-pseudoscalar mixing, thus the five neutral Higgs bosons are no longer states of definite CP parity. In this case, the mass matrix should be diagonalized to obtain mass eigenstates h_i $(i = 1,2,3,4,5)$ whose squared masses $m_{h_i}^2$ $(i = 1,2,3,4,5)$ are the eigenvalues of the mass matrix. These five neutral Higgs bosons are usually numbered such that h_1 is the lightest neutral Higgs boson and h_5 is the heaviest. Hereafter, we work in the explicit CP violation scenario, that is, with $\phi_t \neq 0$.

In our model, the squared masses of the two extra gauge bosons $m_{Z'}^2$ and $m_{Z''}^2$ are obtained as the eigenvalues of the mass matrix for them. The explicit expressions for $m_{Z'}^2$ and $m_{Z''}^2$ are given as

$$\begin{aligned}
m_{Z'}^2 &= \frac{1}{2}(m_Z^2 + m_{Z_1}^2) + \sqrt{m_{Z_1}^2 - m_Z^2)^2 + 4\Delta_1}\ , \\
m_{Z''}^2 &= \frac{1}{2}(m_Z^2 + m_{Z_2}^2) + \sqrt{m_{Z_2}^2 - m_Z^2)^2 + 4\Delta_2}\ ,
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
m_{Z_1}^2 &= \frac{1}{9} g_1^{'2} v^2 (4 - C_{2\theta} + \sqrt{15}\cos 2\beta S_{2\theta}) + \frac{20}{9} g_1^{'2} x_1^2 S_\theta^2 \\
&\quad + \frac{5}{36} g_1^{'2} x_2^2 (8 + 7C_{2\theta} - \sqrt{15} S_{2\theta})\ , \\
m_{Z_2}^2 &= \frac{1}{9} g_1^{''2} v^2 (4 + C_{2\theta} - \sqrt{15}\cos 2\beta S_{2\theta}) + \frac{20}{9} g_1^{''2} x_1^2 C_\theta^2 \\
&\quad + \frac{5}{36} g_1^{''2} x_2^2 (C_\theta + \sqrt{15} S_\theta)^2\ , \\
\Delta_1 &= \frac{1}{3} g_1^{'} m_Z v (\sqrt{5}\cos 2\beta S_\theta + \sqrt{3} C_\theta)\ , \\
\Delta_2 &= \frac{1}{3} g_1^{''} m_Z v (\sqrt{5}\cos 2\beta C_\theta - \sqrt{3} S_\theta)\ .
\end{aligned} \tag{22}$$

The two mixing angles in our model, α_1 between Z and Z' and α_2 between Z and Z'', are expressed as

$$\alpha_i = \frac{1}{2}\tan^{-1}\left(\frac{2\Delta_i}{m_{Z_i}^2 - m_Z^2}\right)\ , \tag{23}$$

for $i = 1, 2$.

III. NUMERICAL ANALYSIS

For the sake of simplicity, we take $g_1^{'} = g_1^{''} = \sqrt{5/3}g_1$ in our numerical analysis, motivated by the gauge coupling unification. We consider the region of the parameter space bounded as $0 < \theta < \pi/2$, $0 < \phi_t < \pi$, $1 < \tan\beta \leq 30$, and $0 < \lambda \leq 0.83$. We assume that the lighter stop quark is heavier than the top quark. We also assume that all of the relevant mass parameters, m_P, m_Q, m_T, and A_t, vary within the range of 100 to 1000 GeV. Note that we employ the mass of the pseudoscalar Higgs boson at the one-loop level in the CP conserving scenario, m_P, instead of A as an input parameter. Further, we use a combined constraint of $\lambda x_1 > 150$ GeV, as the experimental data on the chargino system set the lower bound on the effective μ parameter, $\mu \equiv \lambda x_1$. For the values of x_1 and x_2, we would set their ranges not by hand but by experimental constraints.

There are strong experimental constraints on the mass of the extra neutral gauge boson and the mixing between Z in the SM and the extra neutral gauge boson. Thus, any model with extra neutral gauge bosons, such as our model, should comply with these constraints, whose exact values may dependent on the specific structures of the models. We would like to take in this article that the mixing angles, α_1 and α_2, should be smaller than 3×10^{-3} and the masses of the two extra gauge bosons, $m^2_{Z'}$ and $m^2_{Z''}$, should be larger than 800 GeV.

The experimental constraints on the Higgs sector should also be taken into account in the numerical analysis. The latest experimental analyses tell that the SM Higgs boson lighter than 114.5 GeV is excluded at the 95 % confidence level. This lower bound on the SM Higgs boson mass may be applied to our model by considering the relevant Higgs couplings. Recently, the LEP collaborations reported the model-independent upper bound on $(g_{ZZH}/g^{\rm SM}_{ZZH})^2$ at the 95 % confidence level [16].

First, we determine the values of x_1 and x_2, varying the values of other relevant parameters within their allowed ranges. While the values of the above parameters are chosen by the random number generation method within their respective ranges, the values of x_1 and x_2 are determined in terms of the other parameters by imposing experimental constraints. The result is shown in Fig.1(a), where a distribution of 7125 points is displayed in the (x_1,x_2)-plane. These points are selected among 10^5 random points as they satisfy all of the above experimental constraints. Each point represents a set of parameter values, of which the values of x_1 and x_2 are explicit while others are implicit. It is notable that these selected points are distributed in the area of the (x_1,x_2)-plane where $x_1 + x_2 \geq 2100$ GeV. Some points are scattered at $x_1 \sim 400$ GeV.

Then, we calculate for each point in Fig.1(a) the mass of the lightest neutral Higgs boson in our model. In this way, it is clear that the results are consistent with the relevant parameter ranges as well as the experimental constraints. The result is shown in Fig.1(b). It is quite remarkable that the majority of the points are scattered within the range of $117 \leq m_{h_1} \leq 140$ GeV, while a few of them are distributed where m_{h_1} is as low as 30 GeV. The result of Fig.1(b) suggests that the mass of the lightest neutral Higgs boson in our model is most probably about 130 GeV at the one-loop level.

One may notice some pattern in Fig.1(b). We find that this pattern comes from the experimental constraints on the extra gauge bosons rather than that the experimental

bound on the SM Higgs boson mass. The lower bound on $x_1 + x_2$ is found to arise from the experimental constraints on the masses of the extra neutral gauge bosons. Meanwhile, most of points with $m_{h_1} < 115$ GeV are excluded by the experimental constraints on the SM Higgs boson mass. We also calculate the masses of other neutral Higgs bosons. The results are shown in Figs. 1(c) and (d), where we display the correlation between m_{h_3} and m_{h_2} in Fig. 1(c) and the correlation between m_{h_5} and m_{h_4} in Fig. 1(d). The points in these figures are obtained with the same parameter values as in Figs. 1(a) or (b). Note the clear hierarchy between the masses of the neutral Higgs bosons such that $m_{h_3} > m_{h_2}$ in Fig. 1(c) and $m_{h_5} > m_{h_4}$ in Fig. 1(d). The ranges for the masses of heavier neutral Higgs bosons in our model, estimated using the aforementioned parameter values, are: $100 < m_{h_2} < 997$ GeV, $116 < m_{h_3} < 998$ GeV, $262 < m_{h_4} < 1189$ GeV, and $987 < m_{h_5} < 1536$ GeV, where the upper bounds come from theoretical arguments and the lower bounds come from phenomenological constraints.

Now, we examine the possibility of discovering one of the neutral Higgs bosons in our model in the *pp* collisions at the LHC, where the most dominant process for the Higgs production is the gluon fusion process, with thick QCD backgrounds. The WW fusion process is considered as the next dominant process for the Higgs production, which is relatively cleaner than the gluon fusion process. We would like to focus on the WW fusion process.

We find that the PYTHIA program is useful for calculating the Higgs production mechanism than for other processes, although it has not yet been applied to the CP violation scenario in the MSSM Higgs sector. However, the production cross section of the neutral Higgs bosons in our model with explicit CP violation via the WW fusion process in *pp* collisions is obtained by using the PYTHIA 6.4 program after appropriately modifying the relevant Higgs coupling coefficients [17]. More precisely, we normalize G_{WWH_i}, the WWh_i coupling coefficient of the Higgs coupling to a pair of W bosons, by the corresponding SM Higgs coupling coefficient. We have

$$G_{WWh_i} = (\cos\beta O_{1i} + \sin\beta O_{2i}) \ , \tag{24}$$

where O_{ij} $(i, j = 1, 2, 3, 4, 5)$ are the elements of the orthogonal matrix that diagonalizes the mass matrix for the five neutral Higgs bosons.

Technically, we set the number of events to generate for each point as NEV = 2000. The Higgs coupling coefficient is set by MSTP(4)=1, and the normalized Higgs coupling to a W boson pair is set by PARU(165) = G_{WWh_1}. The factorization scale and the renormalization scale are taken to be the neutral Higgs boson mass, that is, PARP(193) = PMAS(25,1) and PARP(194)= PMAS(25,1). The PDF library of the CTEQ5L is used in our program, MSTP(51)=7, which is the default parton distribution function set for the proton in PYTHIA 6.4. We use MSTP(33)=0 to include the K factor in hard cross sections for parton interactions in PYTHIA 6.4 by default. The WW fusion process for the lightest neutral Higgs boson is set by MSUB(124) = 1.

In this way, we obtain all of σ_{WWh_i} $(i = 1, 2, 3, 4, 5)$, the production cross sections of h_i in our model with explicit CP violation via the WW fusion process in *pp* collisions. They are given as functions of the participating neutral Higgs boson masses. Among the

five production cross sections, we select the largest one, as we are interested in discovering any one of the five neutral Higgs bosons. Thus, we introduce

$$\sigma_{WWh} = \mathrm{MAX}(\sigma_{WWh_1}, \sigma_{WWh_2}, \sigma_{WWh_3}, \sigma_{WWh_4}, \sigma_{WWh_5}) \ . \tag{25}$$

We show our result in Fig. 2, where we plot σ_{WWh} against m_{h_1}. The parameter values for each point are the same as in Fig.1(a) or Fig.1(b). We find that the smallest value for σ_{WWh} is about 1 pb. This implies that at least one of the five neutral Higgs bosons in our model may be produced with its cross section larger than 1 pb. The accumulated integrated luminosity of 30^{-1} fb at the LHC would yield 6000 raw Higgs events, if we allow 20 % for the efficiency and acceptance. Therefore, we expect with relatively strong confidence that at least one of the five neutral Higgs bosons in our model might be produced via the WW fusion process at the LHC, if they exist.

Here, the roles that the exotic quarks take part in are worth mentioning with respect to the Higgs phenomenology of our model. The exotic quarks may inhabit the fundamental 27 representation of E_6, which is the underlying gauge symmetry of our model. In the fundamental 27 representation, 15 components are occupied by the SM matter fields, 4 components by the two Higgs doublets, 2 components by the Higgs singlet, and the remaining 6 components are occupied by the exotic quarks [12-14,18-20].

The form of the superpotential tells us that the exotic quarks may couple to various Higgs fields. They couple directly to the neutral component of the Higgs singlet N_1 and indirectly, through the mixing among the neutral Higgs bosons via the diagonalization matrix, to other neutral Higgs fields. If the masses of the exotic quarks are comparable to the electroweak symmetry breaking or SUSY breaking scales, the low energy SUSY phenomenology might be affected by their presence. The effects of the exotic quarks might appear in the gluon fusion processes for Higgs productions, as well as in the Higgs decay processes. In particular, for example, the Higgs decays into a pair of gluons or photons might receive the effects of the exotic quarks, when the mass of the Higgs boson is below the range where the decay channel into a pair of gauge bosons are not yet open.

However, it is somewhat difficult to predict the amount of the exotic quark effects because it depends on the relevant parameters in a complicated way. The coupling strength of the exotic quarks to the neutral Higgs bosons are weak in general but might be strong, depending on what is the explicit structure of the orthogonal matrix that diagonalizes the mass matrix for the neutral Higgs bosons. Therefore, it would be valuable to study elsewhere a comprehensive research on the effects of the exotic quarks in our model.

IV. CONCLUSIONS

We study a supersymmetric E_6 model with two extra $U(1)$ symmetries besides the SM gauge symmetry, and two neutral Higgs singlets besides two MSSM Higgs doublets. We find that the Higgs sector of our model may generally accommodate a non-trivial complex phase which can cause the scalar-pseudoscalar mixing among the five neutral Higgs bosons, by virtue of radiative corrections due to the top and stop quark loops. Thus, explicit CP violation at the one-loop level is viable in our model.

Numerical analysis shows that there are parameter regions in our model which comply with a number of experimental constraints such as the lower bound on the extra neutral gauge boson masses and the upper bound on the mixing between the extra neutral gauge bosons and the SM neutral gauge boson. Within the allowed parameter regions, we study the behavior of the vacuum expectation values of the two Higgs singlets, x_1 and x_2. We find that they cannot be simultaneously small. The experimental constraints on the extra neutral gauge bosons restrict that $x_1 + x_2$ should be larger than 2100 GeV whereas either one of them may be as small as 400 GeV.

The possibility of discovering one of the five neutral Higgs bosons in our model is examined by calculating the production cross sections using the PYTHIA 6.4 program, where the relevant Higgs couplings are modified suitably. We focus the WW fusion process at the LHC for their productions. We find that at least one of five neutral Higgs bosons can be produced enough via the WW fusion process at the LHC. Thus, we speculate that the present SUSY E_6 model can be tested by the Higgs searches at the LHC.

ACKNOWLEDGMENTS

S. W. Ham thanks his late teacher, Bjong Ro Kim, for learning supersymmetry. He is supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (KRF-2007-341-C00010). He was supported by grant No. KSC-2008-S01-0011 from Korea Institute of Science and Technology Information. This work is supported by Konkuk University in 2007.

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FIGURE CAPTION

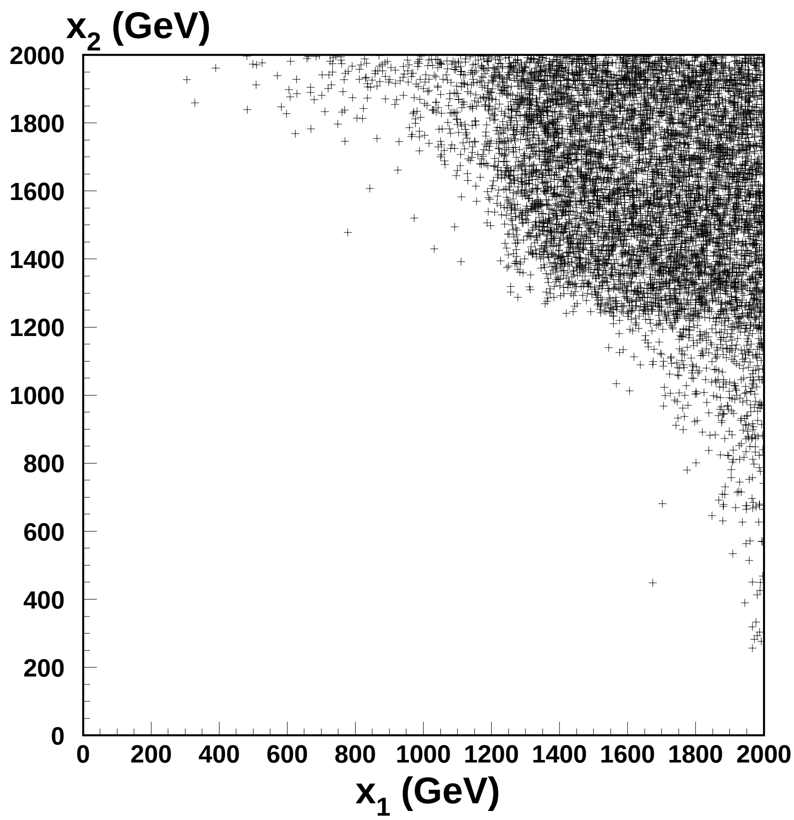
FIG. 1(a). : A distribution of 7125 points in the (x_1, x_2) plane. Each point represents a set of parameter values that satisfies the experimental constraints on the extra neutral gauge boson masses, on their mixings with the SM neutral gauge bosson, and on the SM Higgs boson mass. The values of x_1 and x_2 are explicitly shown, and the other parameters have certain values within their ranges respectively by the random number generation method: $1 < \tan\beta \le 30$, $0 < \lambda \le 0.83$, $0 < \theta < \pi/2$, $0 < \phi_t < \pi$, $100 \le m_A, m_Q, m_T, A_t \le 1000$ GeV.

FIG. 1(b). : The plot of m_{h_1} against $(x_1 + x_2)$. For each of the 7125 points in Fig. 1(a), the mass of the lightest neutral Higgs boson is calculated in terms of the parameter values represented by the point.

FIG. 1(c). : The distribution of 7125 points in the (m_{h_3}, m_{h_2})-plane. They are distributed between $100 < m_{h_2} < 997$ GeV and $116 < m_{h_3} < 998$ GeV, and they satisfy $m_{h_3} > m_{h_2}$. These points are obtained with the same parameter values as in Figs. 1(a) or (b).

FIG. 1(d). : The distribution of 7125 points in the (m_{h_5}, m_{h_4})-plane. They are distributed between $262 < m_{h_4} < 1189$ GeV and $987 < m_{h_5} < 1536$ GeV, and they satisfy $m_{h_5} > m_{h_4}$. These points are obtained with the same parameter values as in Figs. 1(a) or (b).

FIG. 2. : The polt of σ_{WWh} against m_{h_1}. The production cross sections of the five neutral Higgs bosons via WW fusion process in pp collisions are calculated in terms of the parameter values represented by the point, and the largest of them is chosen, for each of the 7125 points in Fig. 1(a).



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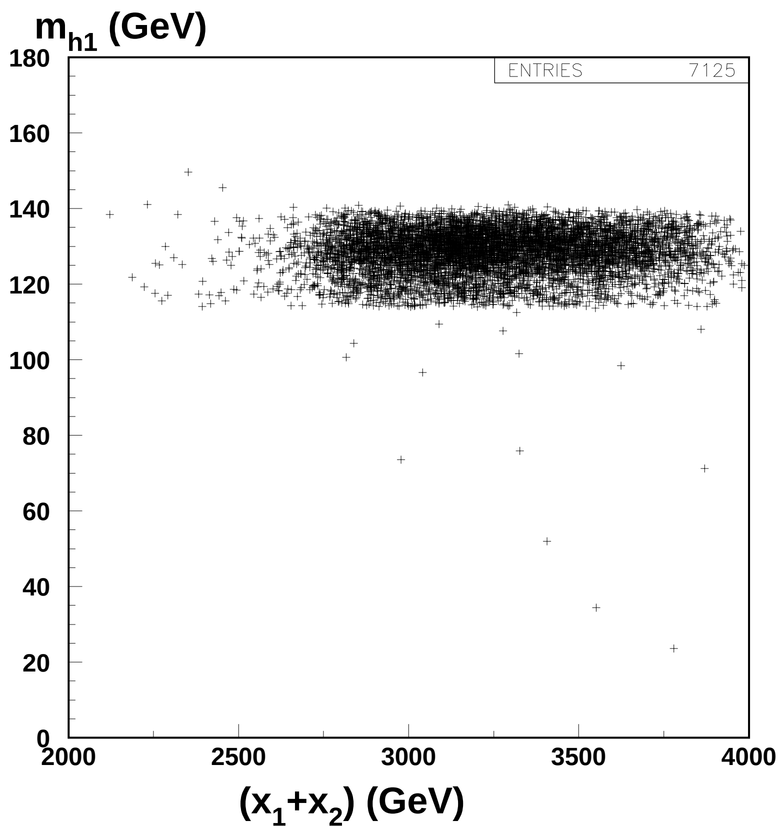

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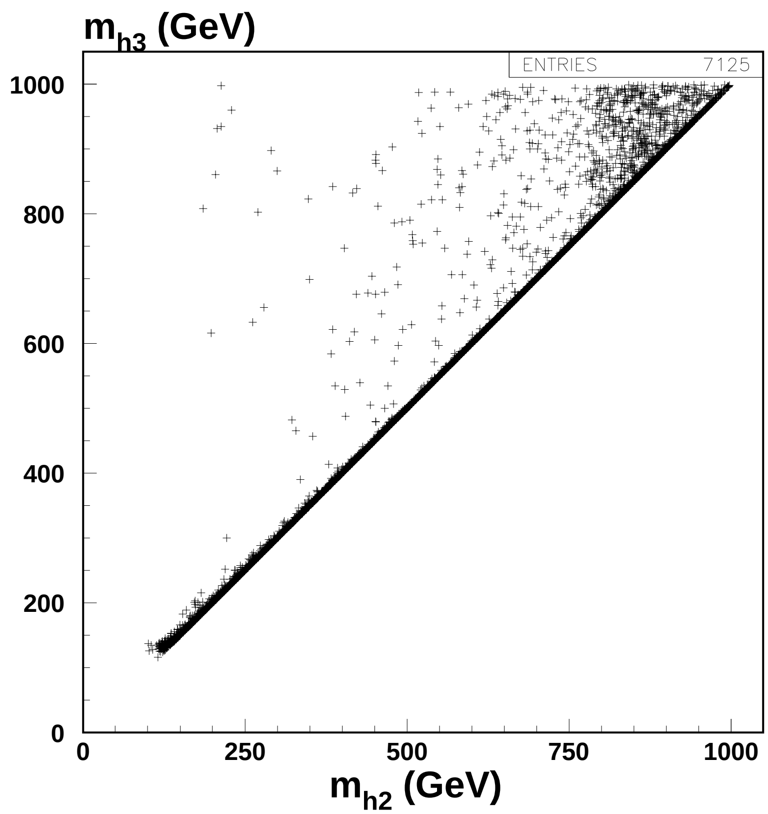

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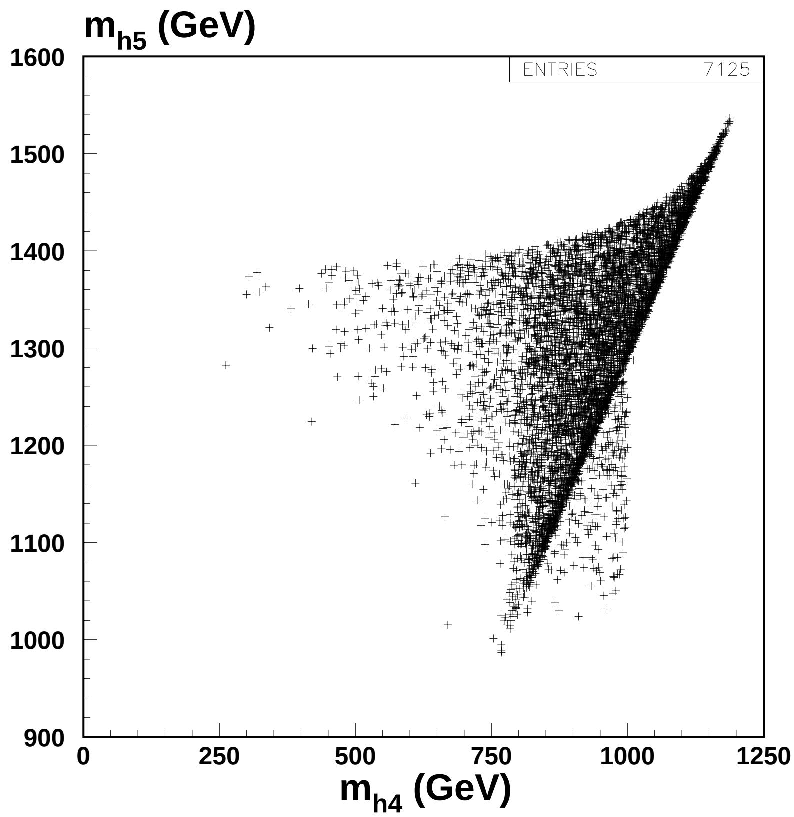

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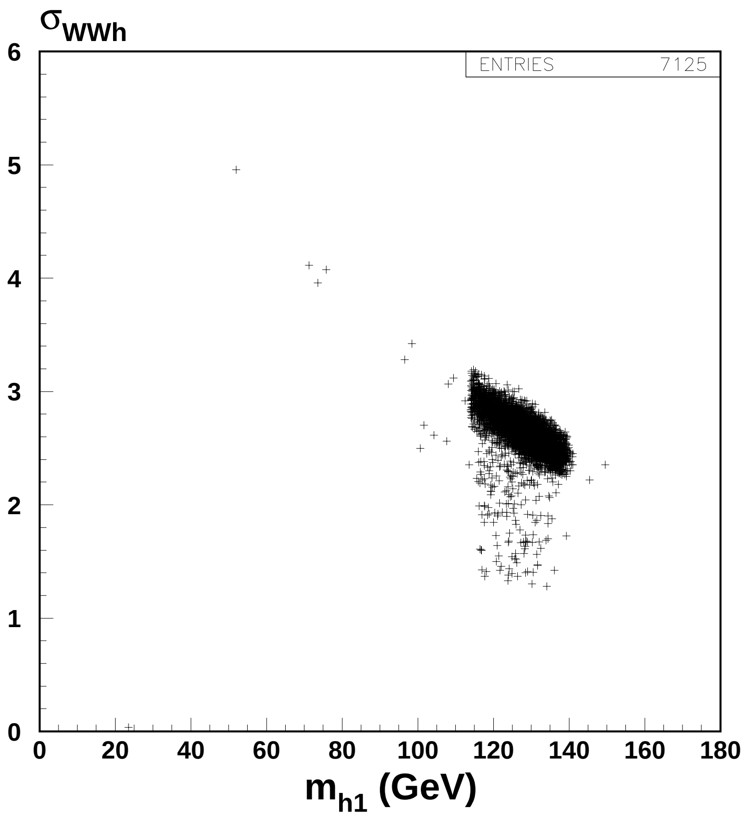

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